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ANALYSIS OF BLOOD FLOW THROUGH MULTI-IRREGULAR SHAPE STENOSED ARTERY

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ABSTRACT

The flow of blood in two-dimensional through the constricted stenosed artery is investigated in this paper. Considered the problem of blood flow in an artery in the presence of multi-irregular shape stenosis. Blood is considered a Newtonian fluid and it is characterized by the generalized form of Navier-Stokes equation. Atherosclerotic plaque can cause severe stenosis in the artery lumen. If a stenosis is present in an artery the normal blood flow is disturbed. A linear approximation of Navies-Stokes equation has been solved with the help of boundary conditions and the results are shown graphically for different flow characteristics. It is found that wall shear stress is increased as the height of stenosis increases. Wall shear stress is increased with axial axis for increasing values of stenosis shape parameter, axial velocity decreases within the stenotic region, the volumetric flow rate and other flow characteristics of blood have been observed.

KEY WORDS

Newtonian fluid, Pressure gradient, Shear stress, Stenosis, Volumetric flow rate.

INTRODUCTION

Atherosclerosis is the major cause of heart attack and stroke. This disease causes a ongoing stenosis of the lumen and hardening of the artery wall because of accumulation of lipids in the intima[19]. The successive build-up of deposits in the arterial wall may form a plaque that protrudes in to the lumen and restricts the blood flow. If the carotid artery (which supplies blood to the brain) is affected then it may cause stroke, when the coronary artery (which supplies blood to the heart) is affected, one may suffer from a heart attack [1,2]. There are also corresponding changes in the forces (shear and normal stresses) exerted by the flowing blood on the plaque surface. The abnormal narrowing in a blood vessel is caused by stenosis. Coronary artery disease is most rare type of heart disease and also causes death. When fatty deposits build up inside the coronary artery then stenosis builds up, it narrows the arteries and reduces the amount of blood flow that enter into the heart. Investigation of the blood flow in a multi-irregular shape stenosed geometry is of our interest because it plays important roles in human vascular diseases. V. P. Srivastava, Shailesh Mishra [11] studied the effects of an overlapping stenosis on non Newtonian blood flow characteristics in a narrow artery. Othman Smadi et al [8] suggested heat transfer and fluid flow analysis of blood flow through multi stenosis arteries with viscous dissipation effect. Daniel N. Riahi, Ranadhir Roy, Sam Cavazos[3] investigated arterial blood flow in the presence of an overlapping stenosis and found the effects of the hematocrit and constriction due the red cells–plasma combination of the variable blood viscosity and the height of the stenosis on these quantities. Rekha Bali, Usha Awasthi[5] investigated

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the effect of magnetic field, height of stenosis, parameter determin- ing the shape of the stenosis on velocity field, volumetric flow rate in stenotic region and wall shear stress at surface of stenosis are obtained and shown graphically. S. Mukhopadhyay, G. C. Layek[6] demonstrate that the flow resistance decreases as the shape of a smooth stenosis changes and maximum resistance is attained in case of a symmetric stenosis.

We propose to study the effect of multi irregular shape stenosis on the flow of blood, when blood is considered as a Newtonian fluid. The aim of the present analysis is to study the flow characteristics of blood in a multi-irregular shape stenosed artery. Here we have taken a new model of stenosed artery. The analytical expression for velocity, volumetric flow rate, pressure gradient and the wall shear stress have been obtained. The numerical solutions for pressure gradient and the wall shear stress have been shown graphically.

MATHEMATICAL MODEL:

Let us consider the flow of blood in a tube having axisymmetric multi-irregular shape stenosis. It is assumed that blood is an incompressible fluid. Two different shapes of stenosis have been taken into account. Consider (r, θ, z) be the cylindrical polar coordinates with the z-axis along the axis of symmetry of the tube, v_z and v_r be the axial and the radial velocity components, respectively, p the pressure, ρ the density, and μ denotes the kinematic viscosity of the fluid. The distance of the starting position of the stenosis from the radial axis is d and in both cases the length of the stenosis is l.

The schematic diagram showing the flow is given by the following figure:

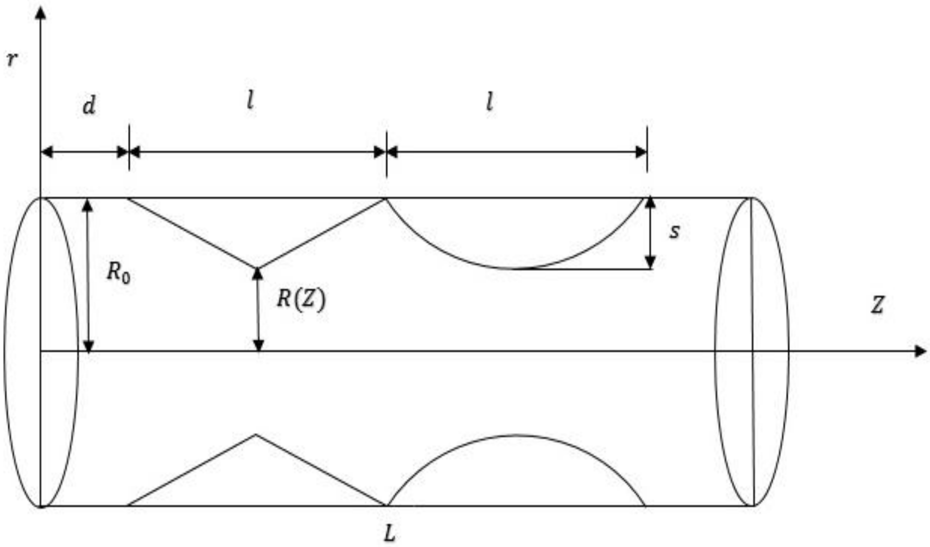

Figure 1 Geometry of a multi-irregular shape stenosed artery

Where $R(z)$ and R_0 are the radius of the artery with and without stenosis, $2l$ is the total length of the stenosis, and s is the maximum height of the stenosis.

The geometry of multi-shape stenosed artery can be expressed as:

$$R(z) = \begin{cases} R_0 - \dfrac{2s}{l}(z-d), & d \le z \le d + \dfrac{l}{2} \\ R_0 + \dfrac{2s}{l}(z-d-l), & d + \dfrac{l}{2} < z \le d + l \\ R_0 - s + \dfrac{4s}{l^2}\left(z - d - \dfrac{3l}{2}\right)^2, & d + l < z \le d + 2l \end{cases}$$

$R0$ Otherwise

(1)

Assumptions and continuity equation. The following assumptions are realistic:

1.There is only one nonzero velocity component, namely that in the direction of flow $v_r = \omega$ at z direction. Thus $v_r = 0, v_\theta = 0$.

2. Gravity acts vertically downwards, so that $g_z = 0$.

3. The axial velocity is independent of the angular location; that is $\frac{\partial v_z}{\partial \theta} = 0$.

From continuity equation we get, $\frac{\partial v_z}{\partial z} = 0$.

Navier –stokes equations in three dimension

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(v_z)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Simplify of these equation we get the equation of steady blood flow,

$$0 = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu \mathrm{r}\frac{\partial w}{\partial r}\right) \tag{2}$$

The boundary conditions of our proposed geometry are:

$$w = 0 \text{ at } r = R(\mathrm{z}) \tag{3}$$

$$\frac{\partial w}{\partial r} = 0 \text{ at } r = 0 \tag{4}$$

Now solving the equation (2) under boundary conditions (3) and (4) we get,

$$w = \left(\frac{1}{2}\frac{dp}{dz}\right)\int \frac{r}{\mu}dr + c$$ (Where c is the integration constant)

$$w = \frac{1}{2}\frac{dp}{dz}\frac{1}{\mu}\frac{r^2}{2} + c$$

$$w = \frac{1}{4}\frac{dp}{dz}\frac{1}{\mu}r^2 + c \tag{5}$$

From equation (2) w = 0 at $r = R(\mathrm{z})$ we get,

$$0=\frac{1}{2}\frac{dp}{dz}\frac{1}{\mu}\frac{R^2(\mathrm{z})}{2}+c$$

$$c=-\frac{1}{4}\frac{dp}{dz}\frac{R^2(\mathrm{z})}{\mu}$$

Now putting the value of c in equation (5) we get the value of w.

$$w=\frac{1}{4}\frac{dp}{dz}\frac{1}{\mu}r^2-\frac{1}{4}\frac{dp}{dz}\frac{R^2(\mathrm{z})}{\mu}$$

$$w=\frac{1}{4}\frac{dp}{dz}\frac{1}{\mu}\left[r^2-R^2(\mathrm{z})\right] \tag{6}$$

The volumetric flow rate is given by,

$$Q=\int_0^R 2\pi rwdr \tag{7}$$

In equation (7) putting the value of w we get,

$$Q=\int_0^R 2\pi r\frac{1}{4\mu}\frac{dp}{dz}\left[r^2-R^2\left(z\right)\right]dr$$

$$Q=-\frac{\pi}{8}\frac{dp}{dz}\frac{R^4(\mathrm{z})}{\mu} \tag{8}$$

From equation (8) we obtain the pressure gradient $\frac{dp}{dz}$, therefore

$$\frac{dp}{dz}=-\frac{8\mu Q}{\pi R^4(\mathrm{z})} \tag{9}$$

Now the shear stress at wall is defined by,

$$\tau_R=\left[\mu\frac{dw}{dr}\right]_{r=R(\mathrm{z})} \tag{10}$$

By using equation (6) and (9) in equation (10) we can find the shear stress at maximum height of the stenosis at $z=d+\frac{l}{2}$, which is as follows

$$w=\frac{1}{4}\frac{dp}{dz}\frac{1}{\mu}\left[r^2-R^2(\mathrm{z})\right]$$

$$=\frac{2Q}{\pi}\left[\frac{1}{R^2(\mathrm{z})}-\frac{r^2}{R^4(\mathrm{z})}\right] \tag{11}$$

Now differentiate equation (11) with respect to r,

$$\frac{dw}{dr}=\frac{2Q}{\pi}\frac{d}{dr}\left[\frac{1}{R^2(\mathrm{z})}-\frac{r^2}{R^4(\mathrm{z})}\right]$$

$$=-\frac{4Qr}{\pi R^4(\mathrm{z})}$$

Applying the boundary condition $w = 0$ at $r = R(z)$, we get

$$\frac{dw}{dr} = -\frac{4QR(\text{z})}{\pi R^4(\text{z})}$$

$$= -\frac{4Q}{\pi R^3(\text{z})}$$

Therefore,

$$\tau_R = \left[\mu\left(-\frac{4Q}{\pi R^3(\text{z})}\right)\right]_{r=R(\text{z})}$$

$$= \left[-\frac{4\mu Q}{\pi R^3(\text{z})}\right]_{r=R(\text{z})}$$

wall stress for stenotic region

$$\tau_{s1} = \left[-\frac{4\mu Q}{\pi\left\{R_0 - \frac{2s}{l}(z-d)\right\}^3}\right] \quad \text{at} \quad d \le z \le d + \frac{l}{2}$$

$$\tau_{s2} = \left[-\frac{4\mu Q}{\pi\left\{R_0 + \frac{2s}{l}(z-d-l)\right\}^3}\right] \quad \text{at} \quad d < z \le d + \frac{l}{2}$$

$$\tau_{s3} = \left[-\frac{4\mu Q}{\pi\left\{R_0 - s + \frac{4s}{l^2}\left(z-d-\frac{3l}{2}\right)\right\}^3}\right] \quad \text{at} \ d + l < z \le d + 2l \tag{12}$$

The dimensionless shear stress $\widehat{\tau_s}$ can be obtained by equation τ_0 and (12),

$$\widehat{\tau_s} = \frac{\tau_s}{\tau_0}$$

We know that $\tau_0 = -\frac{4\mu Q}{\pi {R_0}^3}$, where τ_0 is the wall stress for no stenotic region.

$$\widehat{\tau_{s1}} = \frac{1}{\{1-\frac{2s}{lR_0}(z-d)\}^3}$$

$$\widehat{\tau_{s1}} = \frac{1}{\{1+\frac{2s}{lR_0}(z-d-l)\}^3}$$

$$\widehat{\tau_{s1}} = \frac{1}{\{1-\frac{s}{R_0}+\frac{4s}{l^2 R_0}\left(z-d-\frac{3l}{2}\right)^2\}^3} \tag{13}$$

Here $\widehat{\tau_s}$ is the dimensionless wall shear stress at maximum height of the stenosis that is at $z = d + \frac{l}{2}$.

Now using this condition in equation (13) we get,

$$\widehat{\tau_{s1}} = \frac{1}{(1-\frac{s}{R_0})^3} \tag{14}$$

By the same method we find $\widehat{\tau_s}$ for remaining two range of the stenosed artery.

RESULTS AND DISCUSSIONS

We obtained the analytic expression of blood for different flow characteristics of blood. Now in this section we will discuss the flow characteristics graphically with the use of following numerical data. The values of different parameters are given below:

$$l = 1, d = 1, L = 10, Q = 1, R_0 = 2, \mu = 1, \frac{s}{R_0} \leq 1$$

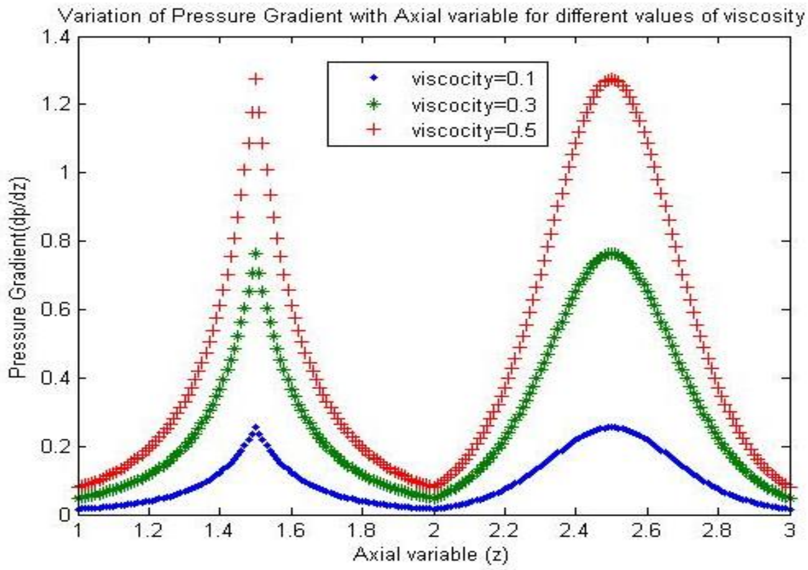

Figure 2: The distribution of Pressure Gradient for different values of viscosity

Here we have shown the graphical representation of pressure gradient. In equation (9) we can see that the value of pressure gradient is dependent upon axial variable. Now from the equations, we put the different z values within range of $R(z)$ that is the equation of the geometry to obtain this relation of Pressure Gradient. Here we have plotted the graphs of pressure gradient for various values of the viscosity of blood. The pressure gradient in stenotic region $\left|\frac{dp}{dz}\right|$ is plotted versus z for different values of viscosity. Figure 2 shows that the pressure gradient increases within the stenotic region and after the stenotic region it is again decreases.

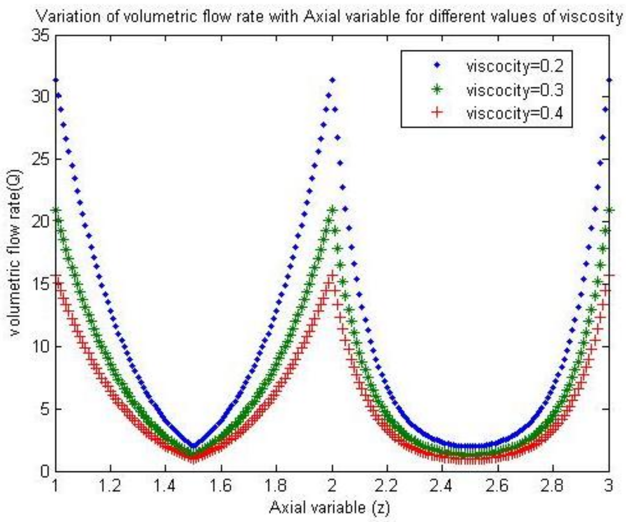

Figure 3: Variation of volumetric flow rate $|Q|$ for different values of viscosity

Here we have showed the graphical representation of volumetric flow rate. In equation (8) we can see that the value of volumetric flow rate is dependent upon axial variable. Here we have taken the pressure gradient as constant ($\frac{dp}{dz} = 1$). The volumetric flow rate of blood in stenotic region $|Q|$ has been plotted versus z for different values of μ. It is observed that the volumetric flow rate of blood decreases as axial variable increases for different values of viscosity. From the graph we can observe that, as viscosity increases the value of volumetric flow rate decreases.

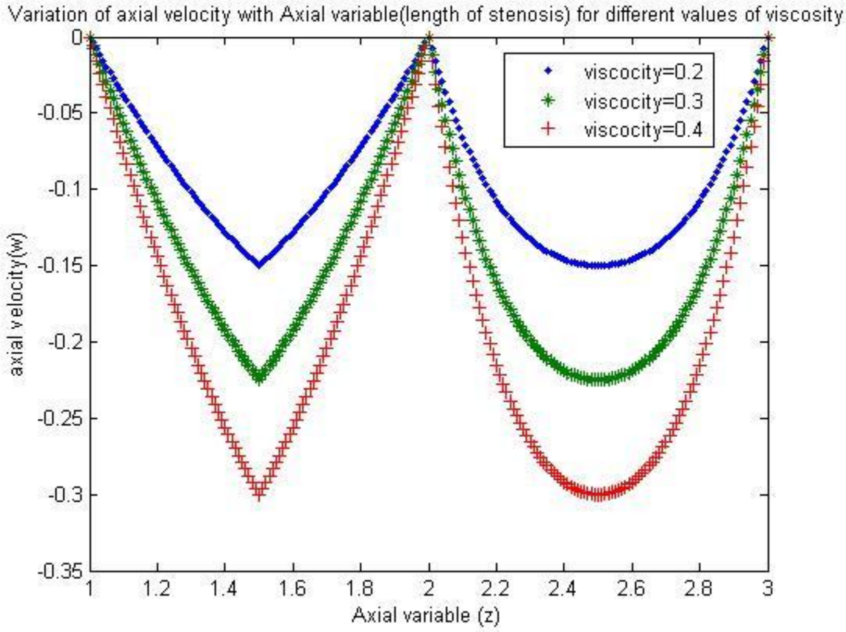

Figure 4: The distribution of axial velocity for different values of viscosity.

Here we have showed the graphical representation of axial velocity. In equation (6) we can see that the axial velocity is dependent up on the length of the stenosis. In equation (6), by putting different $R(z)$ value we can obtain the axial velocity. In this case we assume that the pressure gradient is constant. The axial velocity in stenotic region w plotted versus z for different values of viscosity. Figure 4 shows that the axial velocity decreases within the stenotic region. So it is observed that the axial velocity of blood is decreases in the stenotic region and after the stenotic region it is again increases.

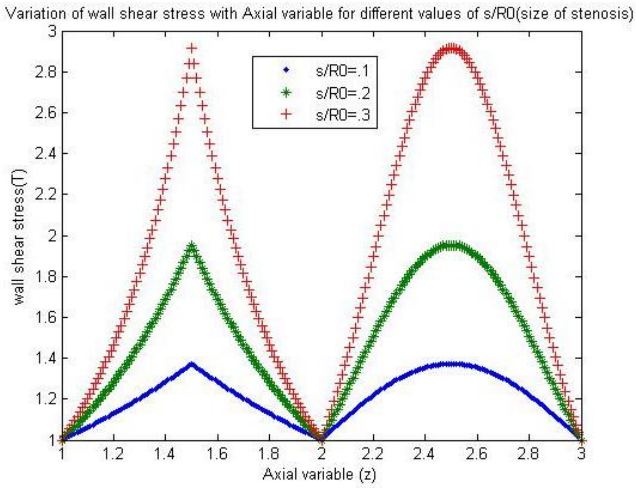

Figure 5: Variation of wall shear stress for different values of $\frac{s}{R_0}$

The wall shear stress can be obtained from the equation (13). Here we have displayed the graphical representation of wall shear stress. The shear stress at the surface of stenosis has been plotted versus axial variable for different values of that is the size of the stenosis. Different values are 0.1, 0.2 and 0.3

respectively. The wall shear stress in stenotic region is plotted versus axial variable for different values of stenosis size. Figure 5 shows that the wall shear stress increases as the size of the stenosis increases with respect to axial variable in the stenotic region.

CONCLUSION

A theoretical and analytical study of blood flow through multi-shape stenosed artery has been carried out. The numerical experiment is helpful for biologist and medical practitioners to analyse the effect of blood flow in presence of multi irregular shaped stenosed artery. The multi-irregular shape stenosis have important effect on flow than single-regular shape stenosis.

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